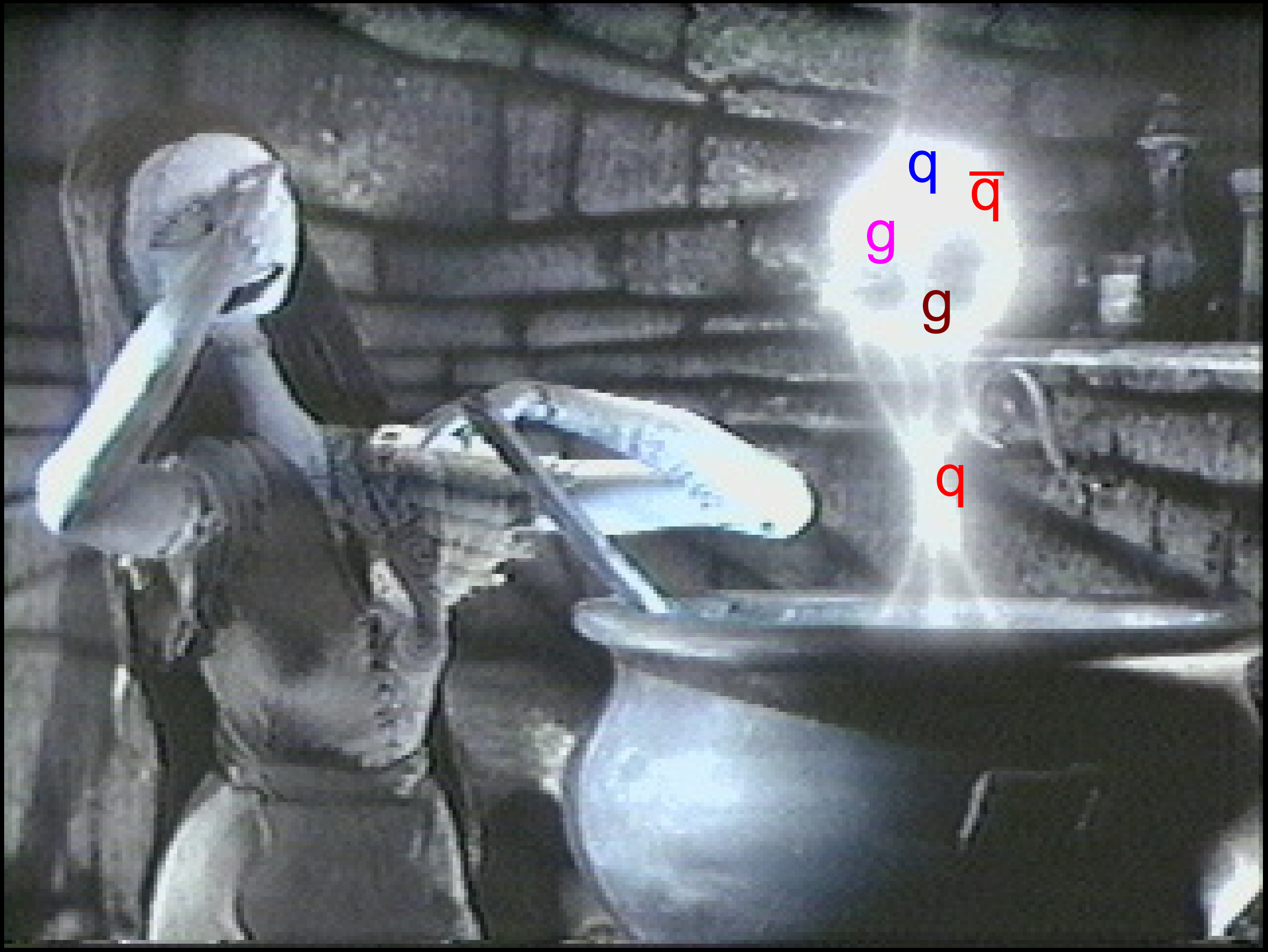
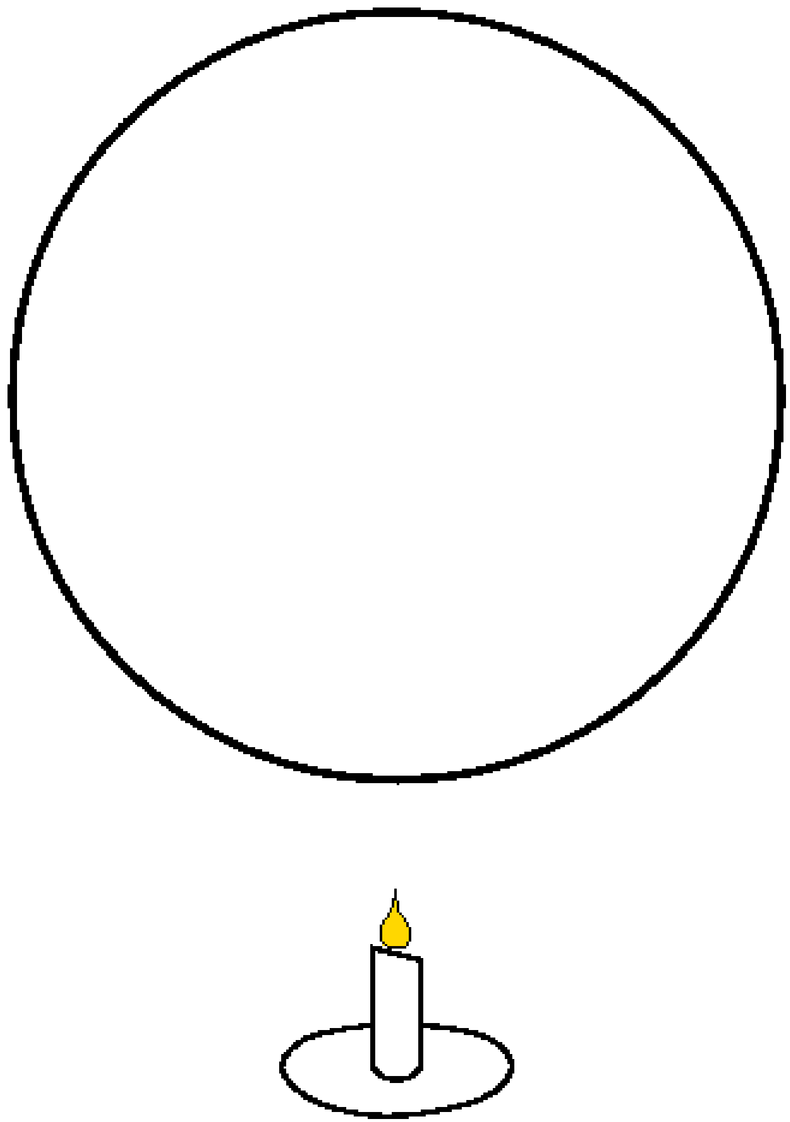
Weakly Coupled Quark Gluon Plasmas

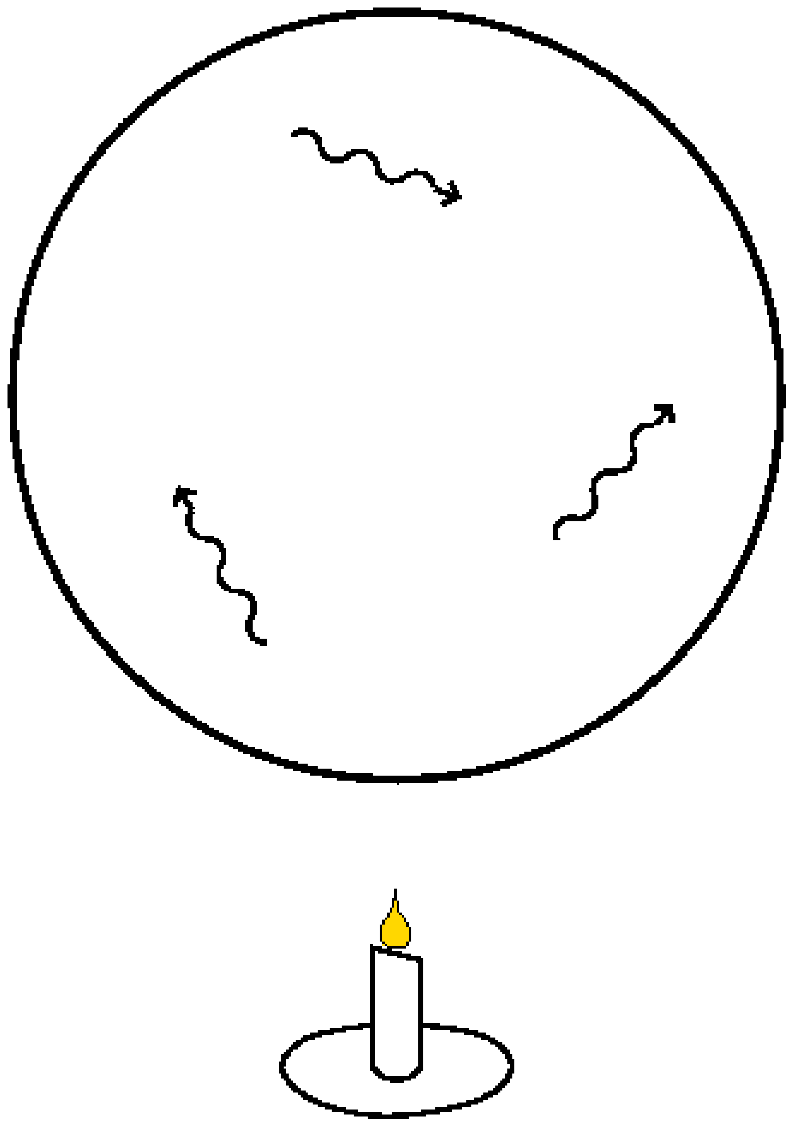


Peter Arnold, University of Virginia

Notice

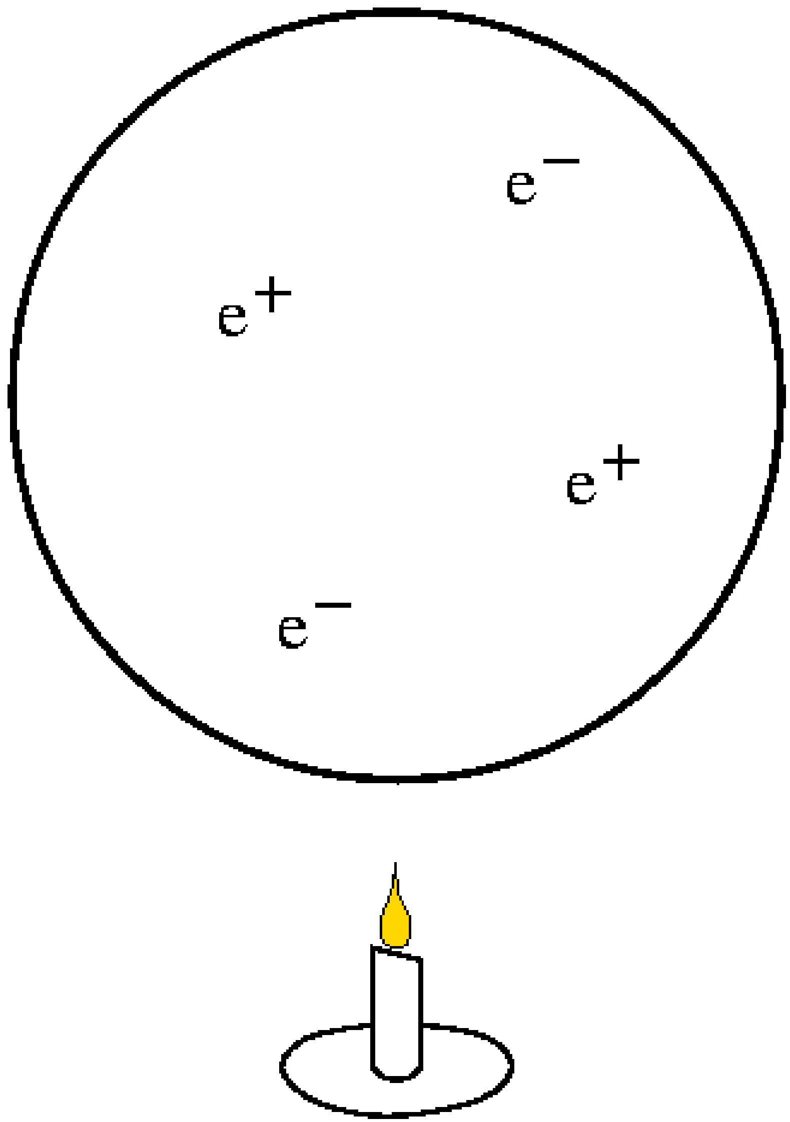
This presentation has been carefully purged of anything that might cause embarassment to the laboratory.

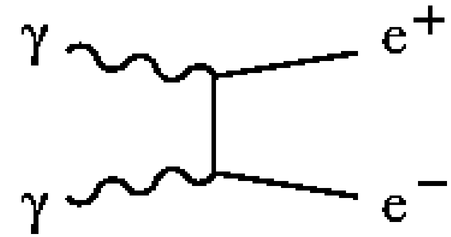




blackbody radiation

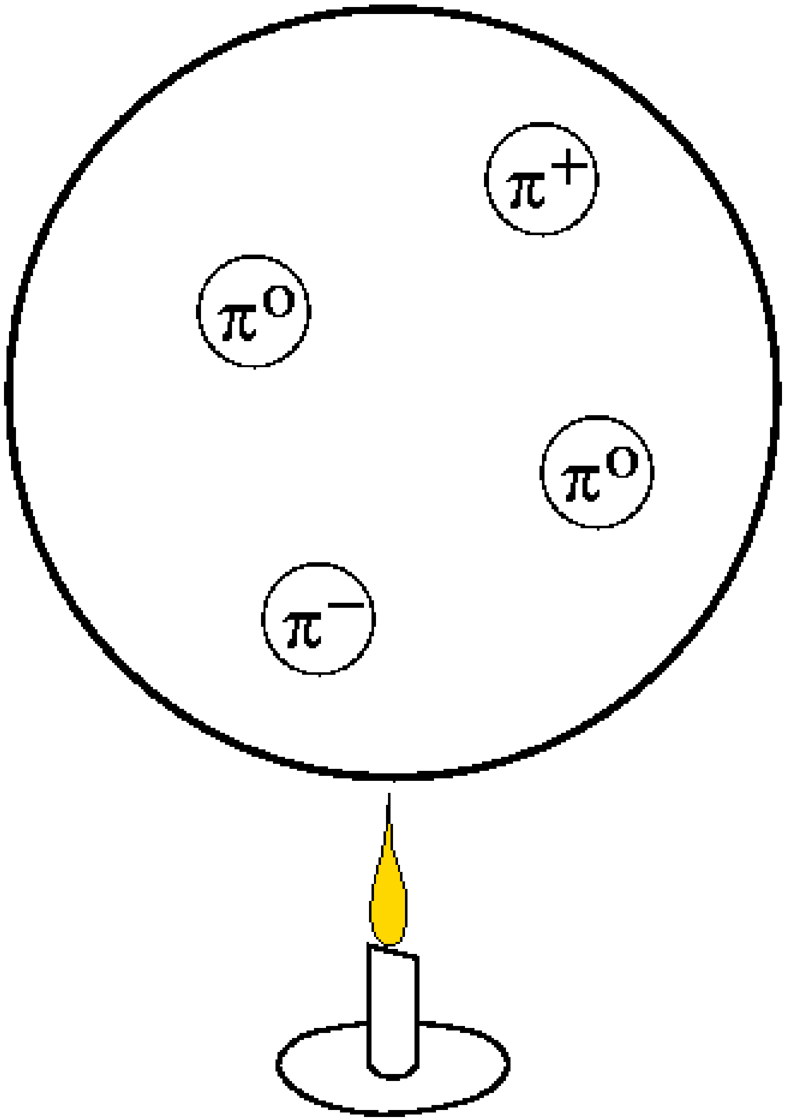
$k_B T \sim 0.5$ MeV

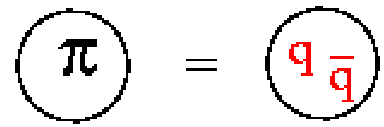




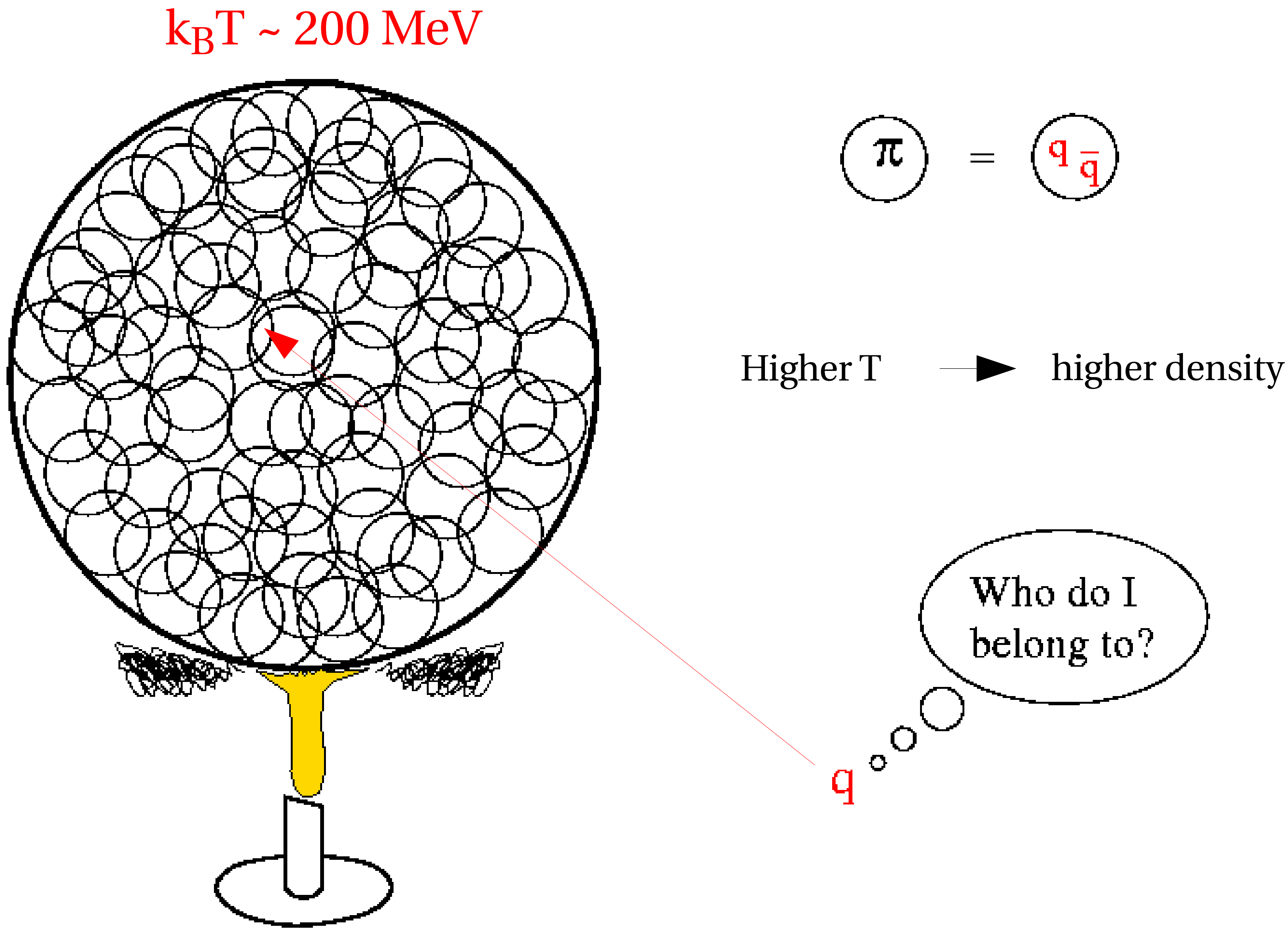
$$\gamma\,\gamma \rightleftharpoons e^+ e^-$$

$k_BT \sim 50$ MeV



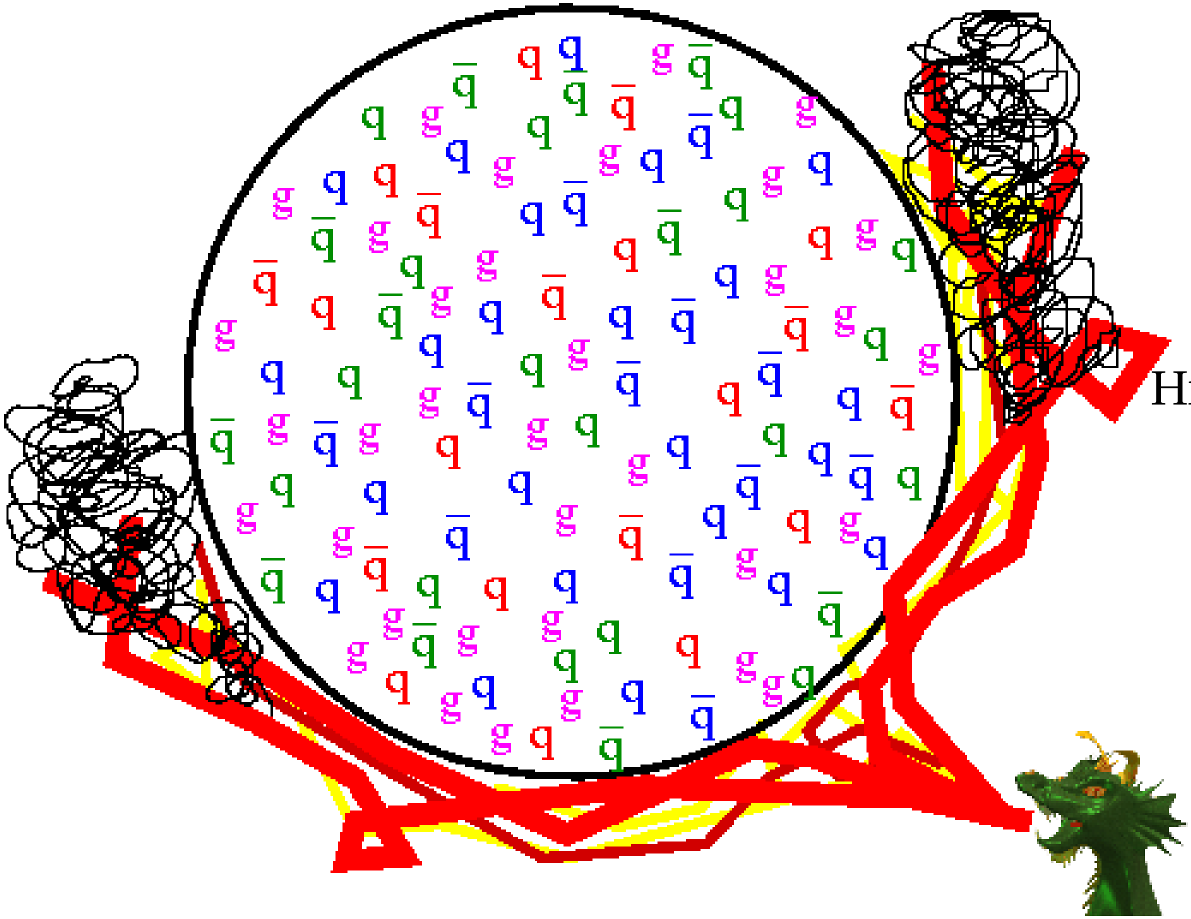


Higher T $\rightarrow$ higher density



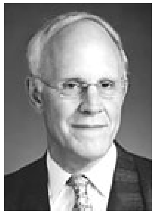
$k_B T \sim 200$ MeV
$\pi = q\bar{q}$
Higher T → higher density
Who do I belong to?
q

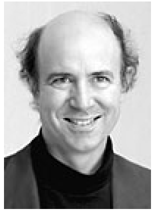
$k_B T >> 200$ MeV

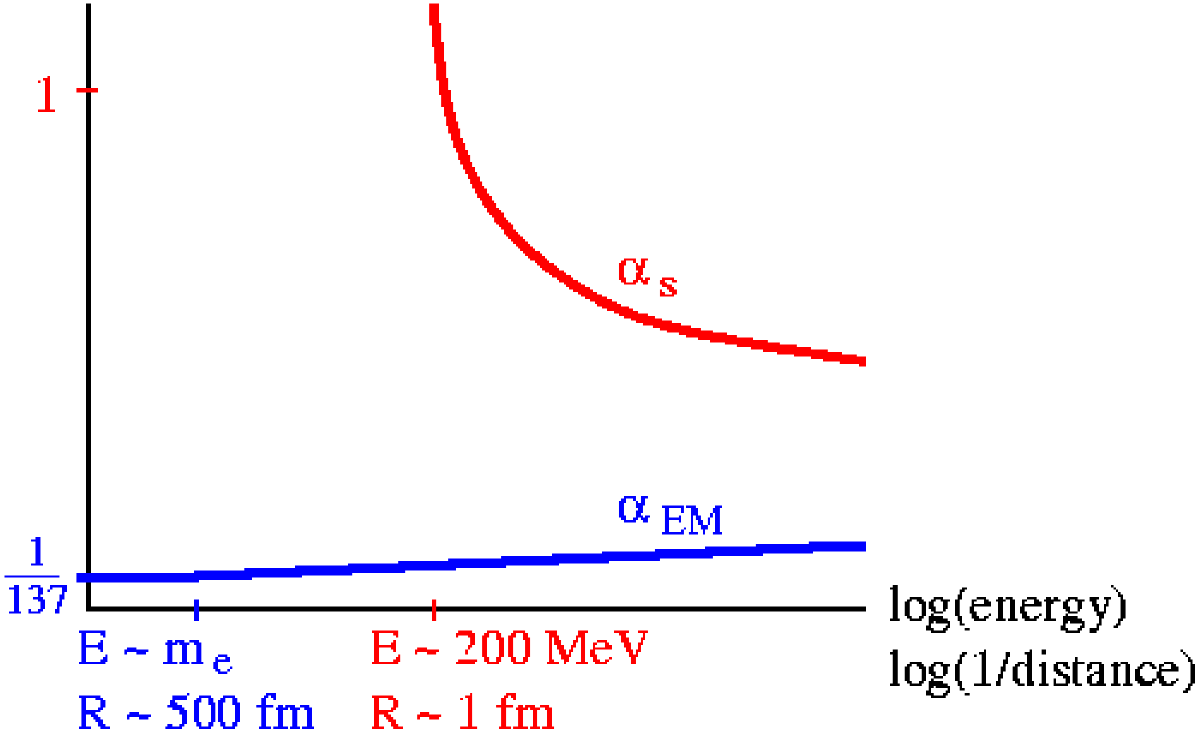


Higher T → higher density

Also: Asymptotic Freedom







$$\alpha \equiv \frac{e^2}{4\pi\hbar c}$$

Higher temperature → smaller coupling α_s

Why bother with weak coupling?

It's one of the few limits where we can do calculations from first principles.

- Lattice simulations

  imaginary time: difficult to apply to real-time response.

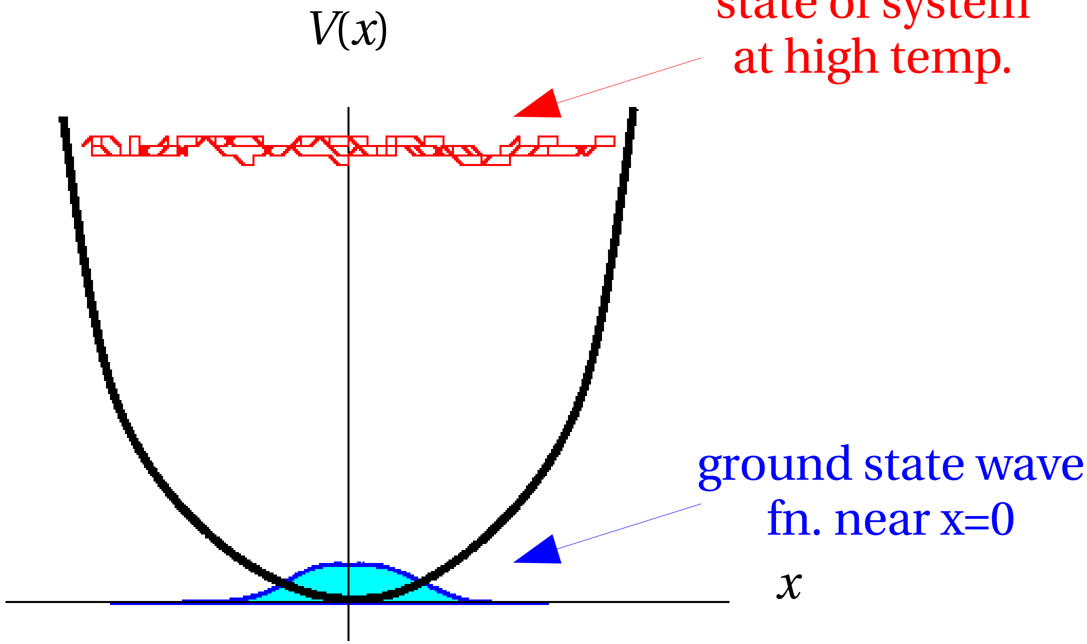
- Consider T very large so that running coupling $\alpha_s(T)$ is small.

- Change the theory (add lots of supersymmetry, take # colors to infinity) and then use AdS/CFT methods to study limit of *really* big coupling.

Isn't weak coupling easy?

Counter-example

$$V(x) \sim \omega_0^2 x^2 + g^2 x^4$$



Note 1: problems with perturbation theory if T high enough.

Note 2: For fixed T, $\omega_0 \to 0$ ==> non-perturbative.
For gauge theory, $\omega_k \sim k \to 0$ ==> non-perturbative.

Moral: small coupling expansion not the same as the perturbative expansion.

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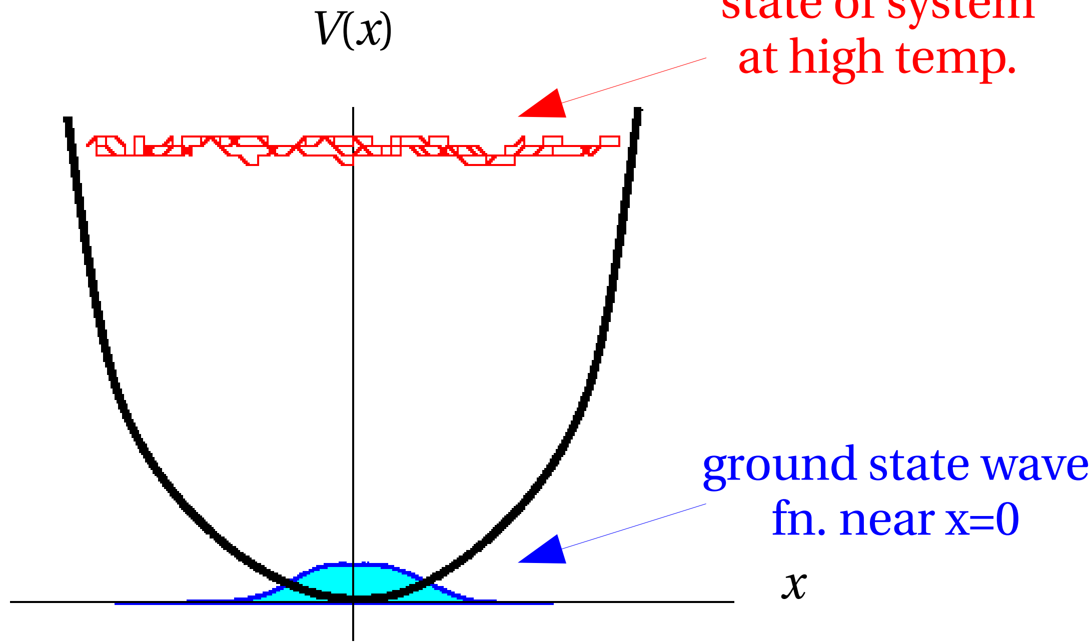

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Example: $P = \# T^4 [1 + \# g^2 + \# g^3 + g^4(\# \ln g + \#) + \# g^5 + g^6(\# \ln g + \#) + \cdots]$

non-perturbative

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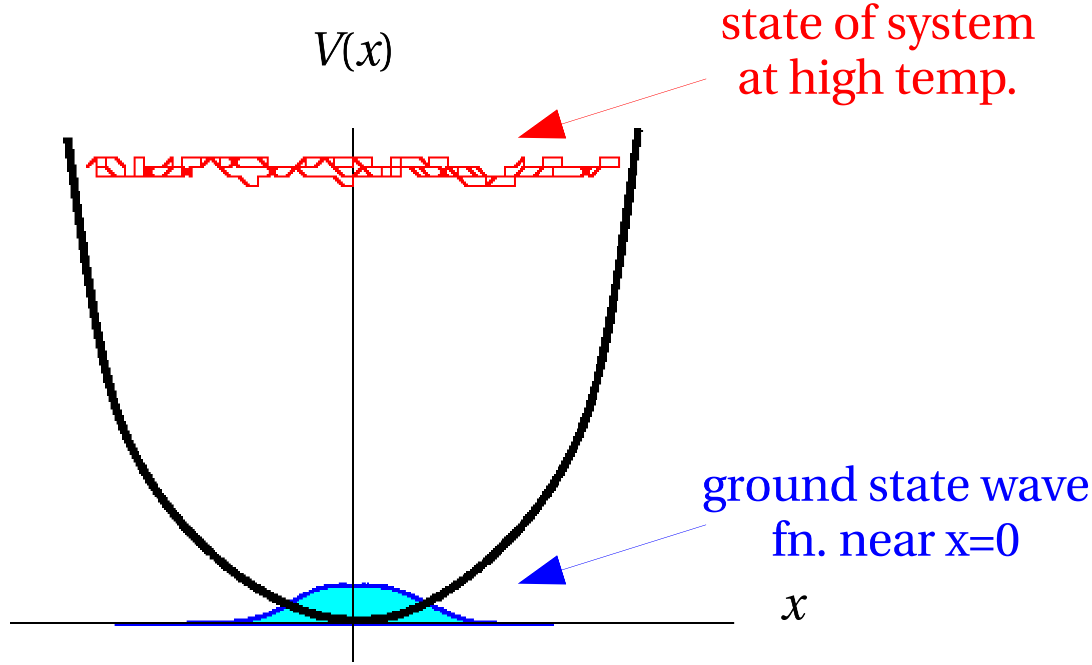

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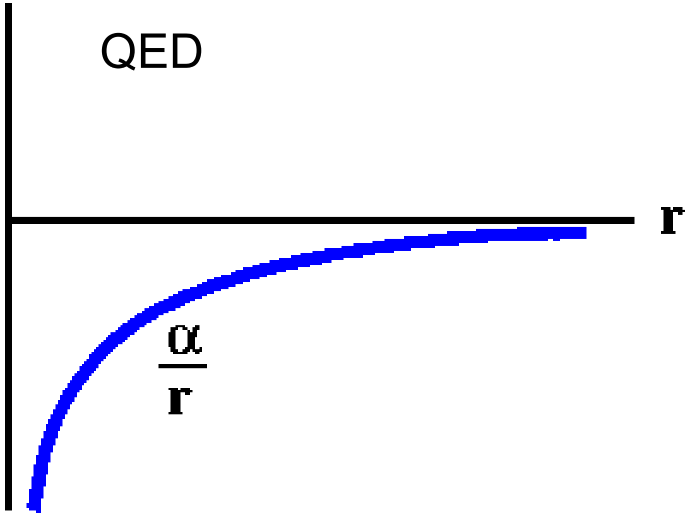
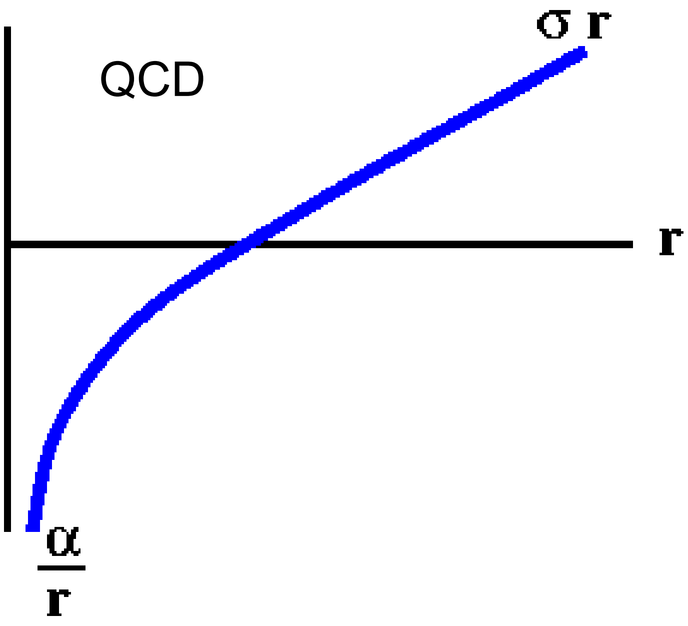
non-perturbative

units: $\hbar = c = k_B = 1$

Deconfinement as Debye Screening

Potential energy between 2 charges in vacuum

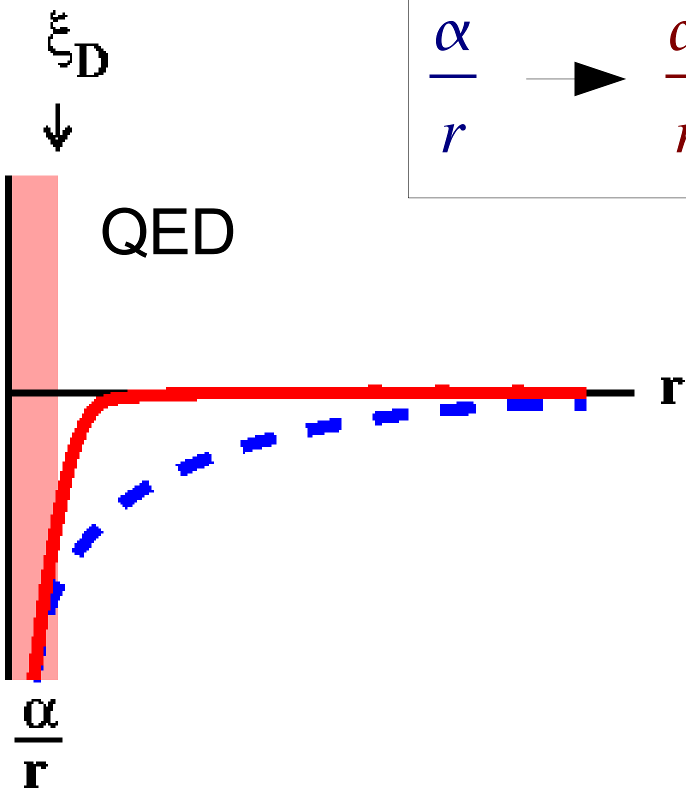


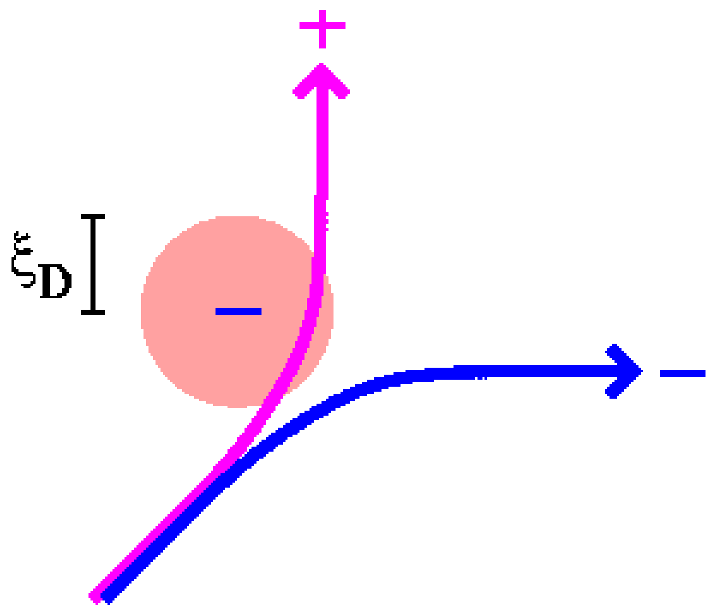


Deconfinement as Debye Screening

In a medium with free charges:

$$\frac{\alpha}{r} \longrightarrow \frac{\alpha}{r} e^{-r/\xi_D}$$





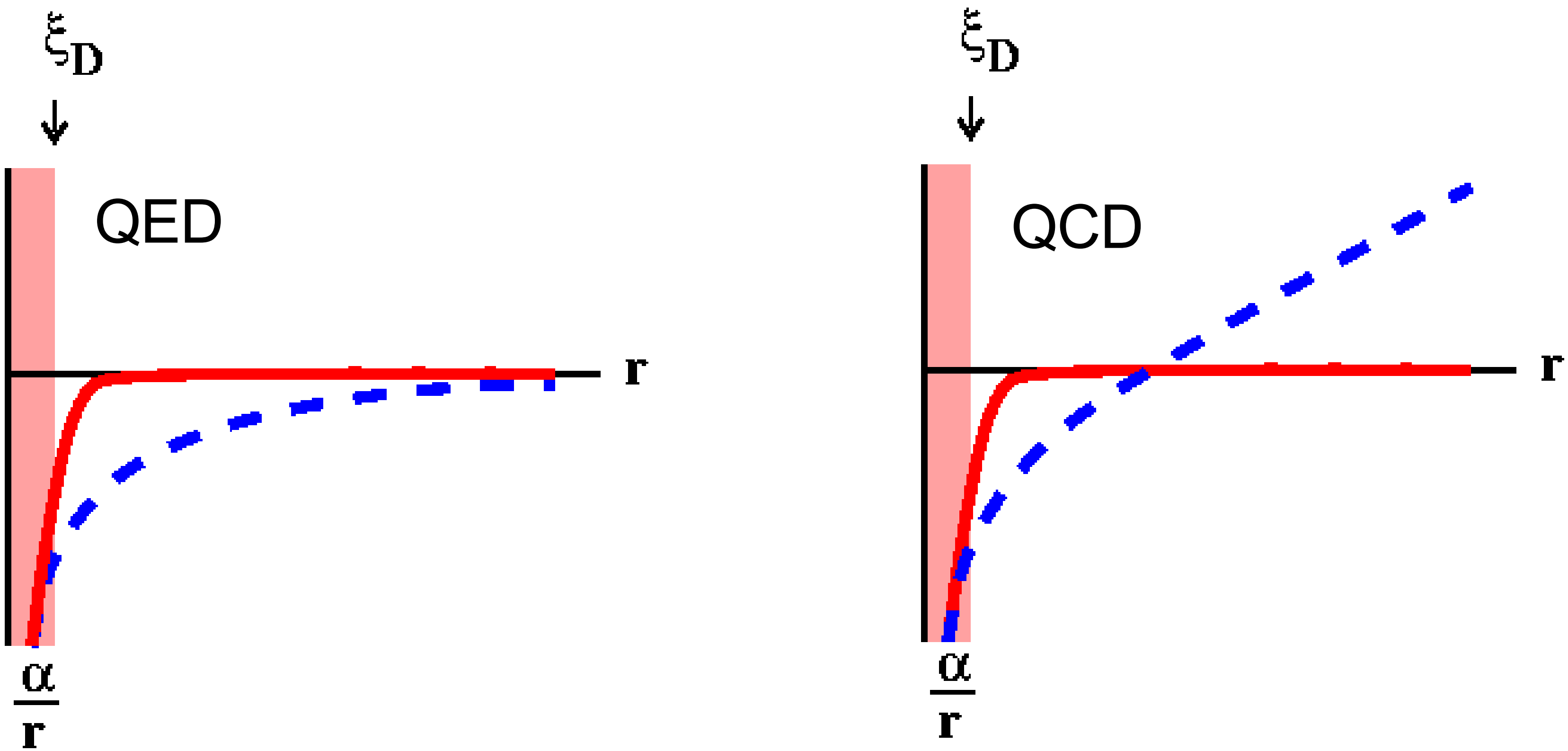
Debye screening length: $\xi_D \sim \left(\dfrac{T}{e^2 n}\right)^{1/2}$

ultra-relativistic: $n \sim T^3 \longrightarrow \xi_D \sim \dfrac{1}{eT}$

Higher temperature $\longrightarrow$ smaller Debye radius

Deconfinement as Debye Screening

In a medium with free charges:



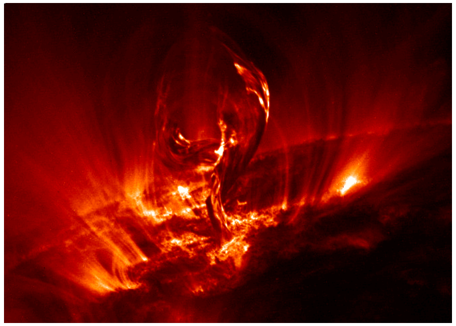
Higher temperature → smaller Debye radius

The Debye effect screens electric fields. In contrast:

Magnetic fields are <u>not</u> screened in a plasma.

So

QED: magnetic forces are still long range



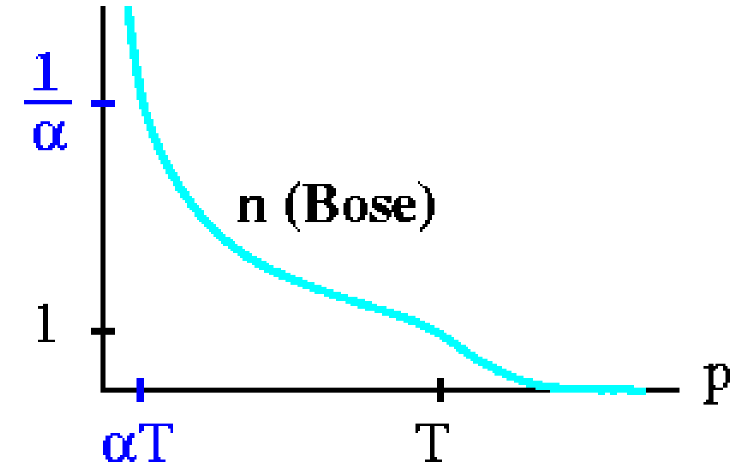
QCD: could there be confinement of colored currents?

→ no long range colored B fields?

Version for particle theorists: Do spatial Wilson loops still have area-law behavior?

YES, and at very short distances too!

$$n_{\text{Bose}} = \frac{1}{e^{\beta E}-1} \to \frac{T}{E} \quad \text{as} \quad E \to 0$$

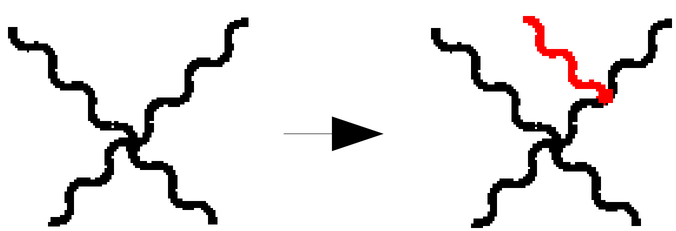


For massless bosons,

$$E \sim p \sim \alpha T \quad \longrightarrow \quad n_{\text{Bose}} \sim \frac{1}{\alpha}$$

Photons don't directly interact with each other, but gluons do.

Result: Perturbation theory breaks down for gluons with $p \sim \alpha T$.



costs $|g|^2 \sim \alpha$ for extra interaction

$n_{\text{Bose}} \sim \frac{1}{\alpha}$ for density of extra gluons

---

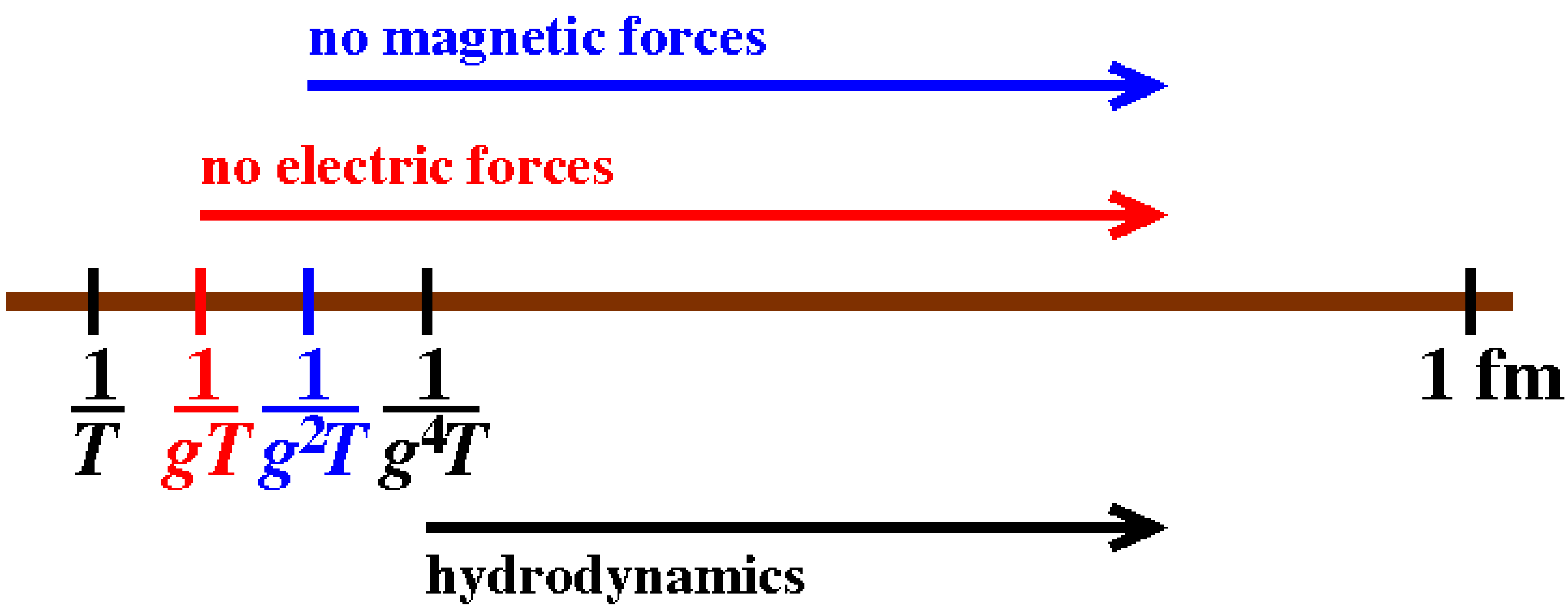
1 total

Summary

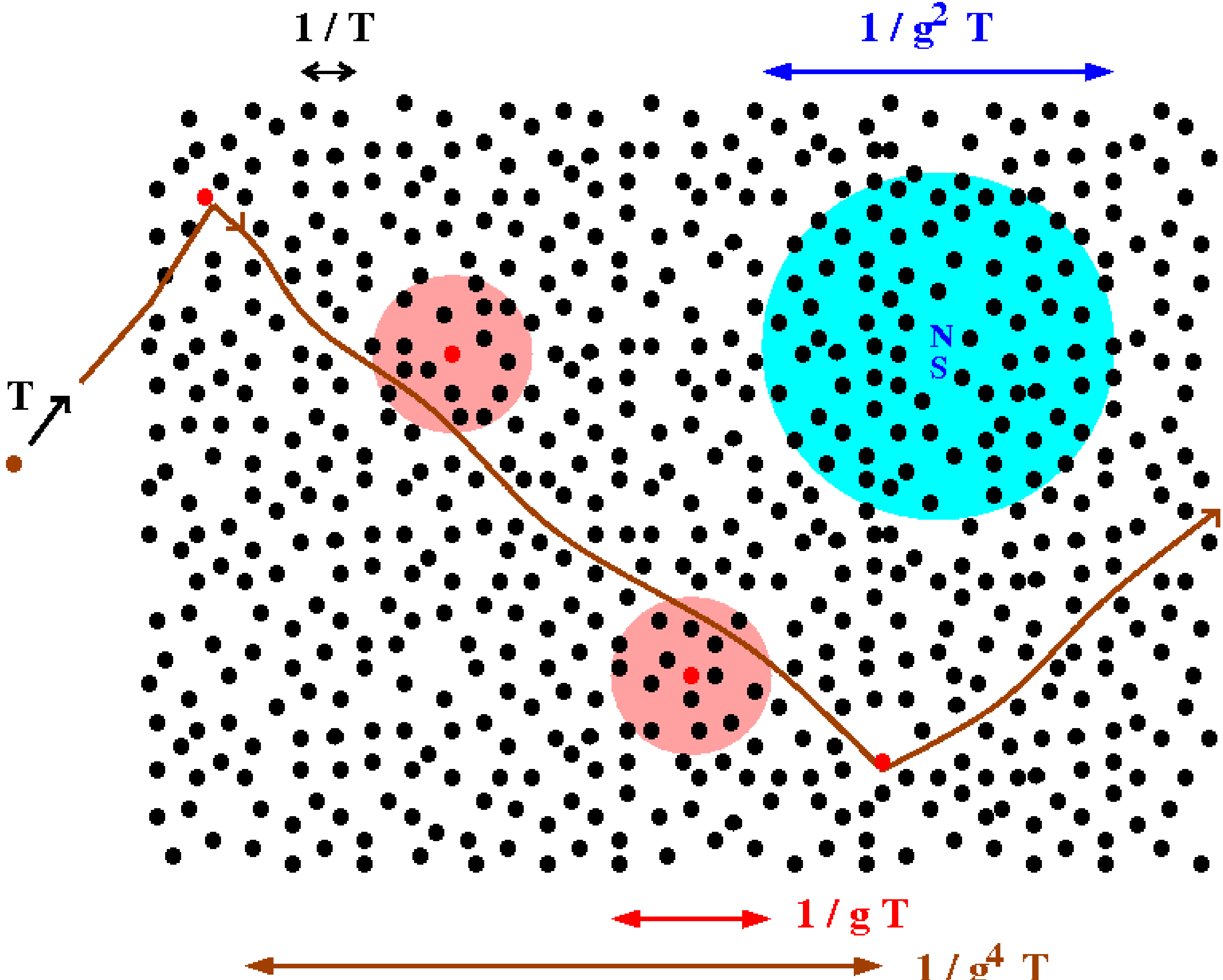
Note: "g" is QCD analog of "e"

electric screening at $\xi_D \sim \dfrac{1}{gT}$ ⟶ no charge confinement

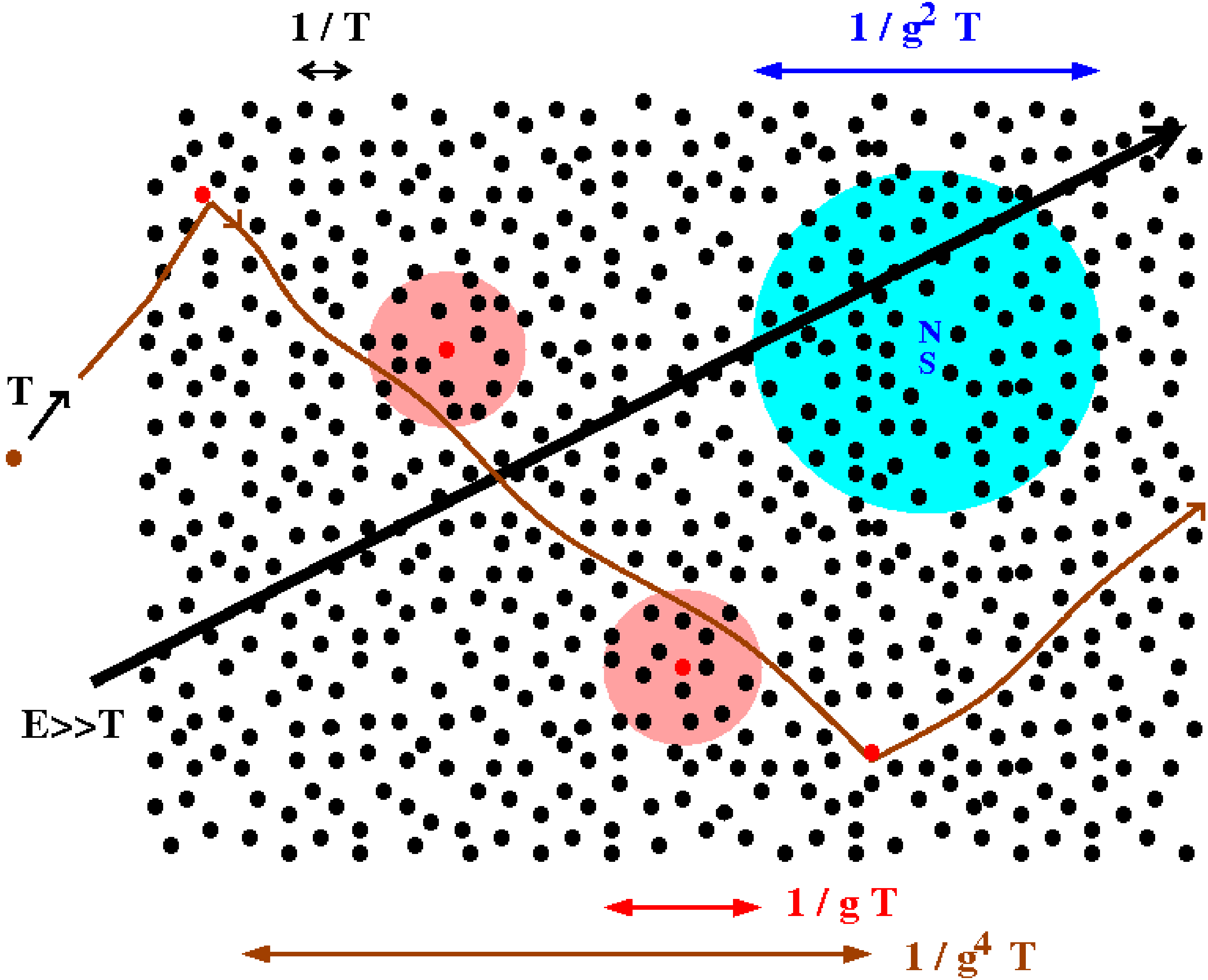
no traditional magnetic screening ⟶ current confinement at $\dfrac{1}{g^2 T}$



Long distance physics is hydrodynamics, not colored MHD.



1 / T
1 / g2 T
T
N
S
1 / g T
1 / g4 T



1 / T
1 / g² T
N
S
T
E>>T
1 / g T
1 / g⁴ T

Landau-Pomeranchuk-Migdal (LPM) effect

<u>What is the LPM Effect?</u>

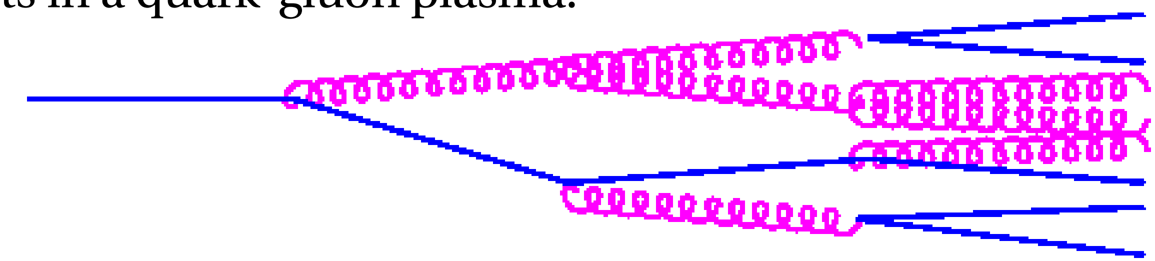
A coherence effect that complicates calculations of bremsstrahlung or pair production when a very high energy particle scatters from a medium.

<u>Places it comes up in QED</u>

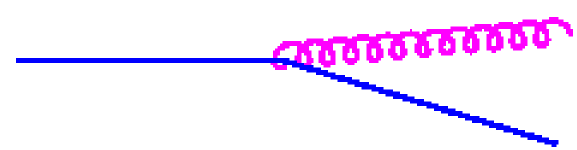
- Very high energy cosmic rays showering in the atmosphere.
- Certain beam dump experiments designed to measure the LPM effect.

<u>Places it comes up in QCD</u>

- Energy loss of high energy jets in a quark-gluon plasma.

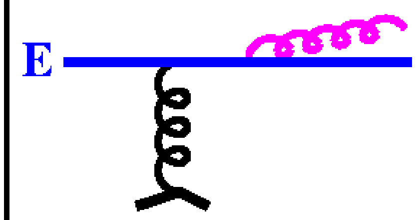


- Complete leading-order calculations of the viscosity and other transport coefficients of a weakly-coupled quark-gluon plasma.



The LPM Effect

Naively

brem rate ~ $n\sigma v$ ~ (density of scatterers) × $\Big|$ 

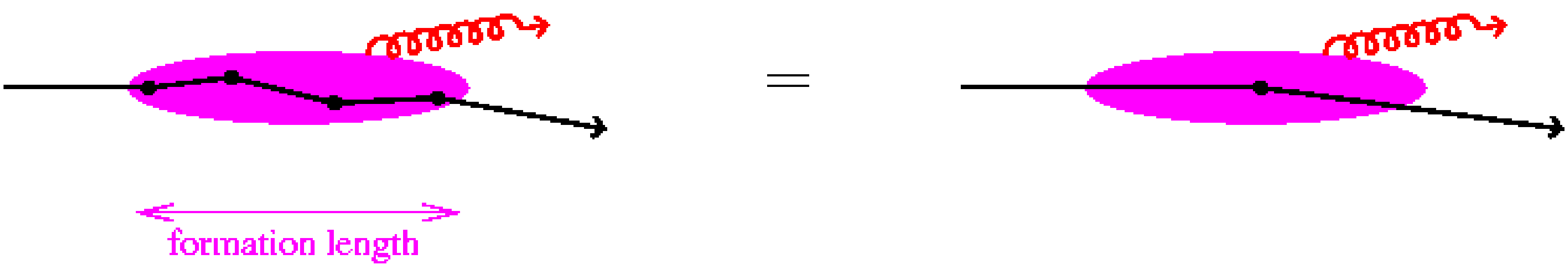
$\Big|^{\mathbf{2}}$ × 1

Problem

At very high energy,

probabilities of brem from successive scatterings no longer independent;

brem from several successive (small angle) collisions not very different from brem from one collision.



$$\text{formation length} \propto \sqrt{E}$$

Result: a reduction of the naive brem rate.

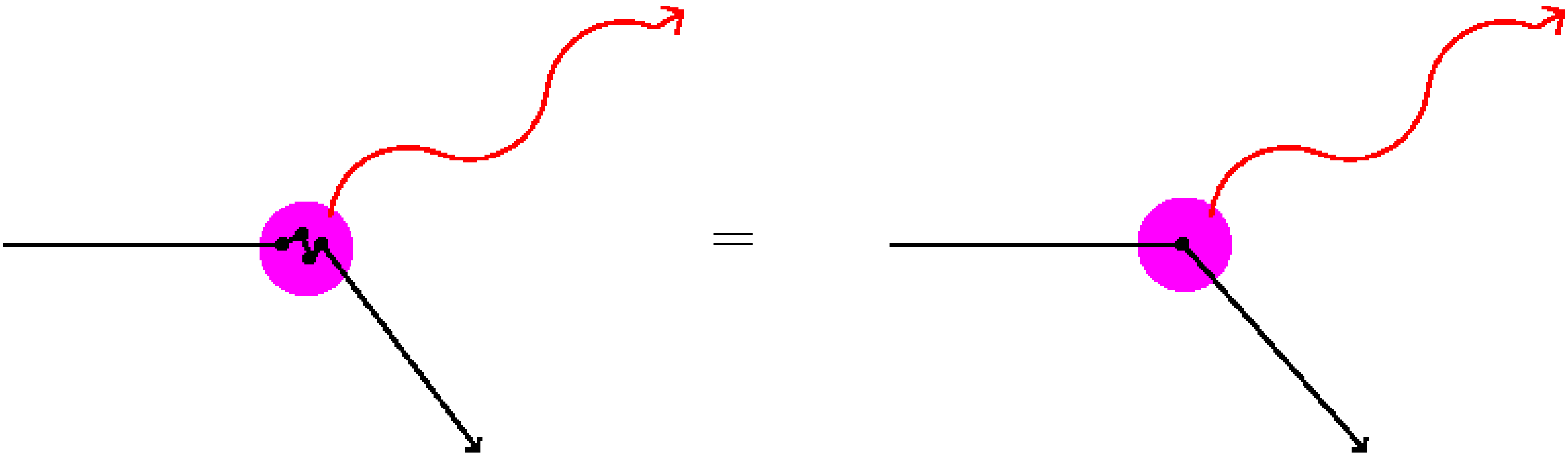
Example: stopping distance
(in a infinite medium)

If LPM effect ignored: stopping distance $\propto \ln E$

Actual result (weak coupling): stopping distance $\propto \left(\frac{E}{\ln E}\right)^{1/2}$

The LPM Effect (QED)

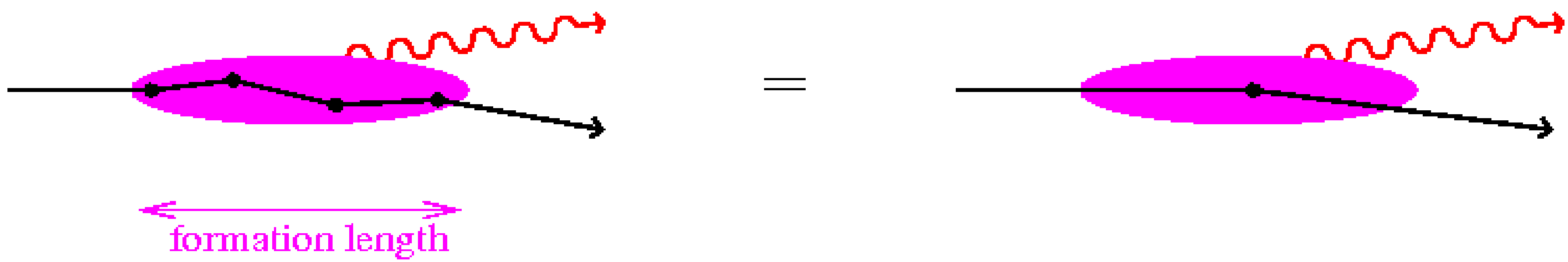
Warm-up: Recall that light cannot resolve details smaller than its wavelength.



[Photon emission from different scatterings have same phase → coherent.]

Now: Just Lorentz boost above picture by a lot!

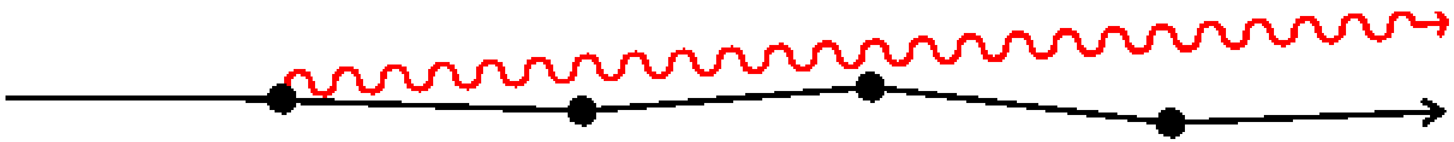
The LPM Effect (QED)



Note: (1) **bigger *E*** requires bigger boost → more time dilation → **longer formation length**

(2) big boost → this process is **very collinear.**

An alternative picture



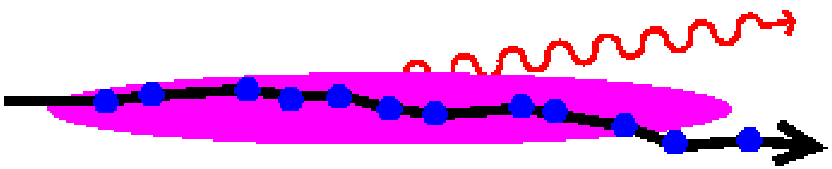
versus

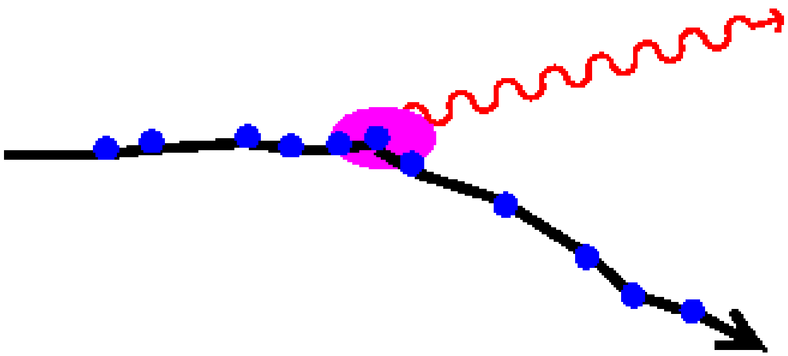
Are these two possibilities in phase? Or does the interference average to zero?

IN PHASE if (i) everything is nearly collinear ✓
(ii) particle and photon have nearly same velocity ✓ (*speed of light*)

The important point:

The more collinear the underlying scattering, the longer the formation time.





Note: the formation length

depends on the net angular deflection during the formation length, which ***depends on*** the formation length

[Self-consistency $\rightarrow$ standard parametric formulas for formation length.]

The LPM Effect (QCD)

There is a qualitative difference for ***soft*** bremsstrahlung.:

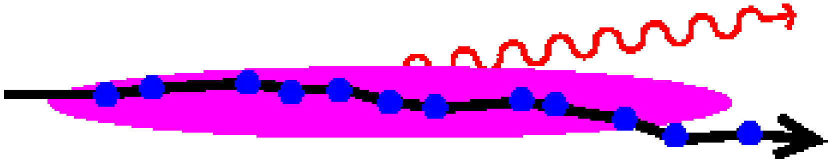
QED

Softer brem photon $\rightarrow$ longer wavelength
$\rightarrow$ less resolution
$\rightarrow$ more LPM suppression

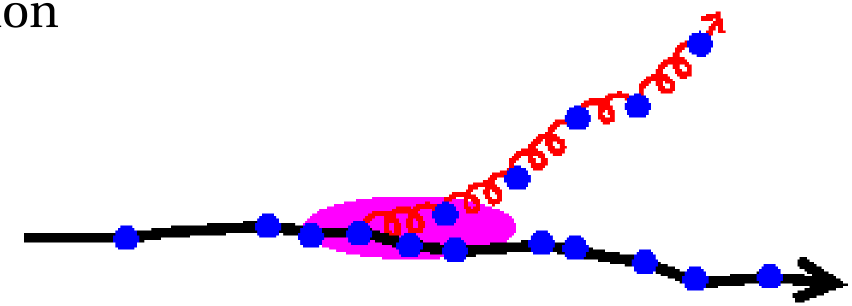
QCD

Unlike a brem photon, a brem gluon can easily scatter from the medium.

Softer brem gluon $\rightarrow$ easier for brem gluon to scatter
$\rightarrow$ less collinearity
$\rightarrow$ less LPM suppression

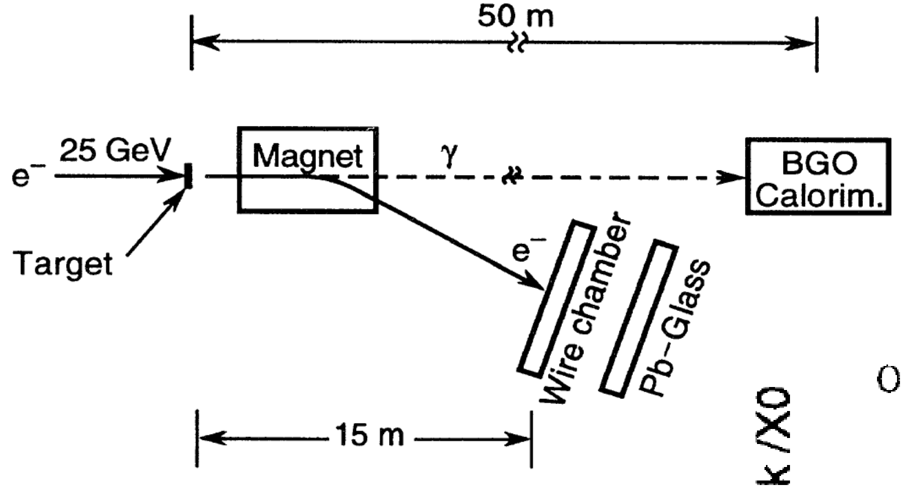


vs.



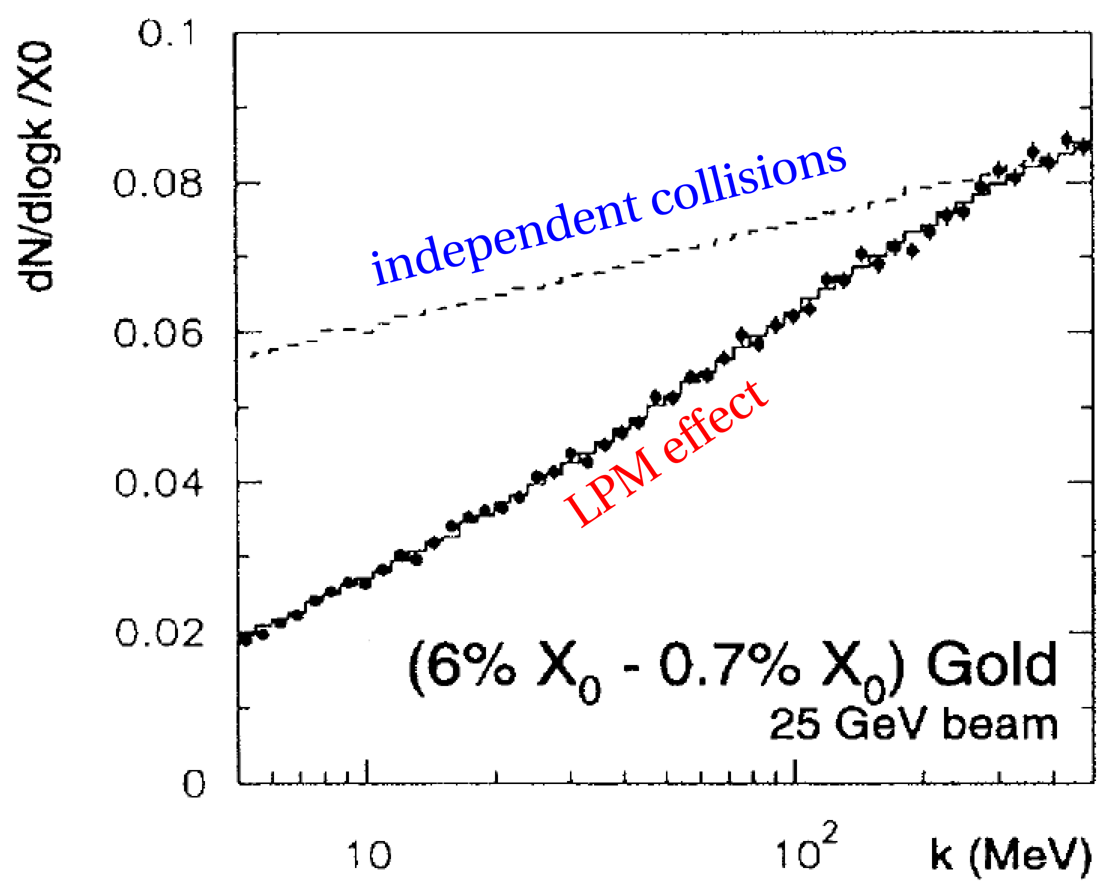
Upshot: Soft brem more important in QCD than in QED (for high-*E* particles in a medium)

Experimental Measurement of LPM (QED)

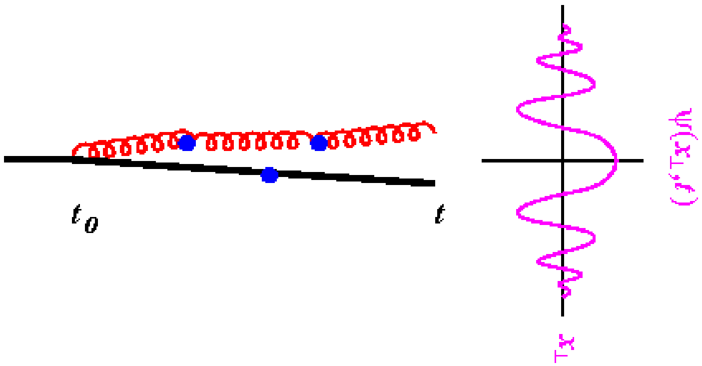


SLAC E-146

Phys. Rev. Lett. **75** (1995) 2949.

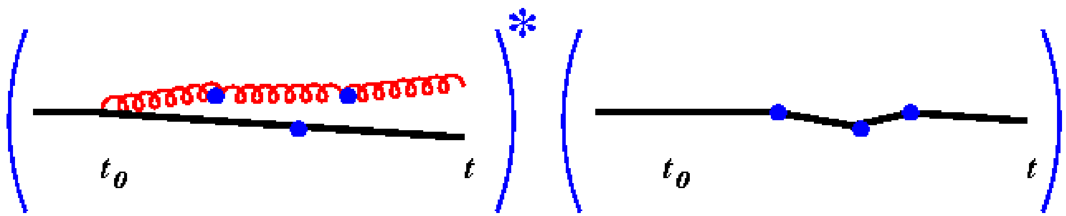


2-dimensional Quantum Mechanics



QM in the transverse plane $\vec{x}_\perp$

But what we really need is the time evolution of the interference



[QED: Migdal '56]

random-averaged over the locations and types of scatterers in the plasma.
Evolution of this interference is described by a 2-dimensional Schrödinger eq. with

$$H(t) = \frac{p_\perp^2}{2\mathcal{M}} - i\,\Gamma(x_\perp, t)$$

for non-uniform media

non-Hermitian

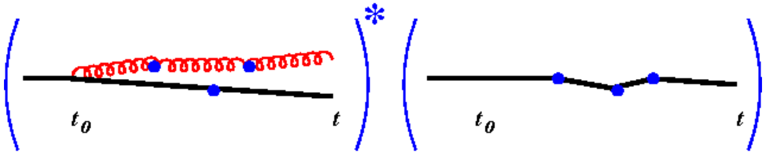
$Ex(1-x)$

What assumptions have been made?

screening length << mean free path

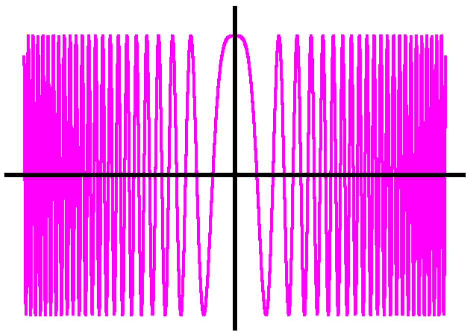
$$\frac{\#}{gT} \ll \frac{\#}{g^2 T}$$

$$H(t) = \frac{p_\perp^2}{2\mathcal{M}} - i\,\Gamma(x_\perp, t)$$



<u>Numerically a bit tricky</u>

$\delta(x_\perp)$ at t_0 $\xrightarrow[t]{}$



for $\Gamma=0$, for example

<u>Harmonic Oscillator approximation</u>

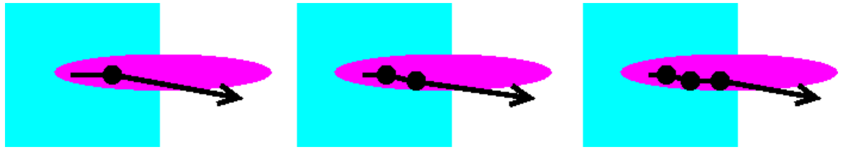
$$H(t) = \frac{p_\perp^2}{2\mathcal{M}} - i\hat{q}x_\perp^2$$

Turns out to apply to thick media

at very large energies: $\ln(E/T) >> 1$

<u>QM Perturbation theory</u> (the opacity expansion)

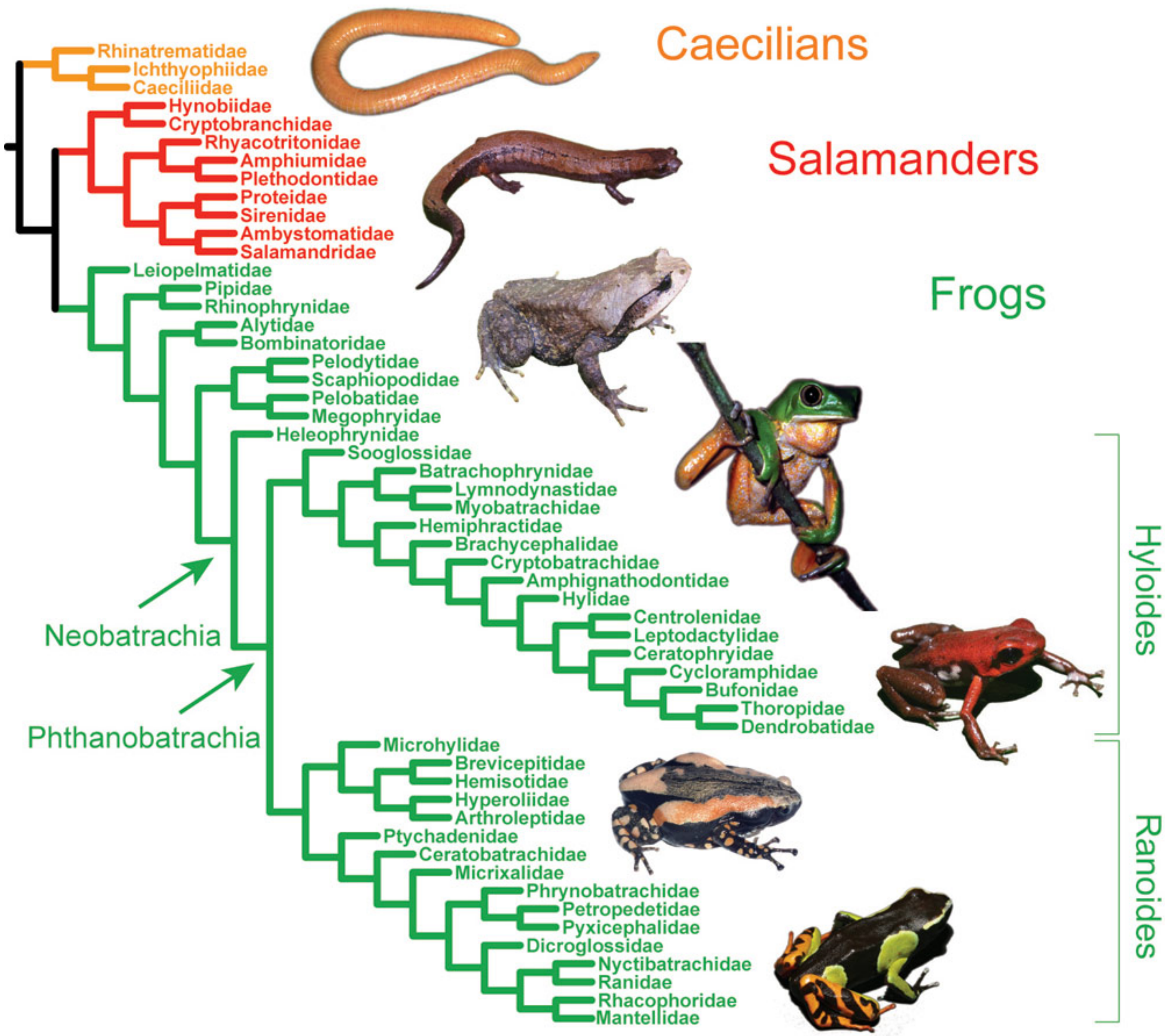
$$H_0 + \delta H(t) = \frac{p_\perp^2}{2\mathcal{M}} - i\,\Gamma(x_\perp, t)$$



Applies to thin media

(but needn't be as thin as you might think)

Taxonomy of Jet Quenching Formalisms



BDMPS-Z/ASW

Taxonomy of Jet Quenching Formalisms

<u>But</u> I'm going to restrict attention to

- effectively massless partons (no heavy quark jets)
- methods based on the preceding formalism

Apologies in particular to the "higher twist" (HT) jet quenching members of the community.

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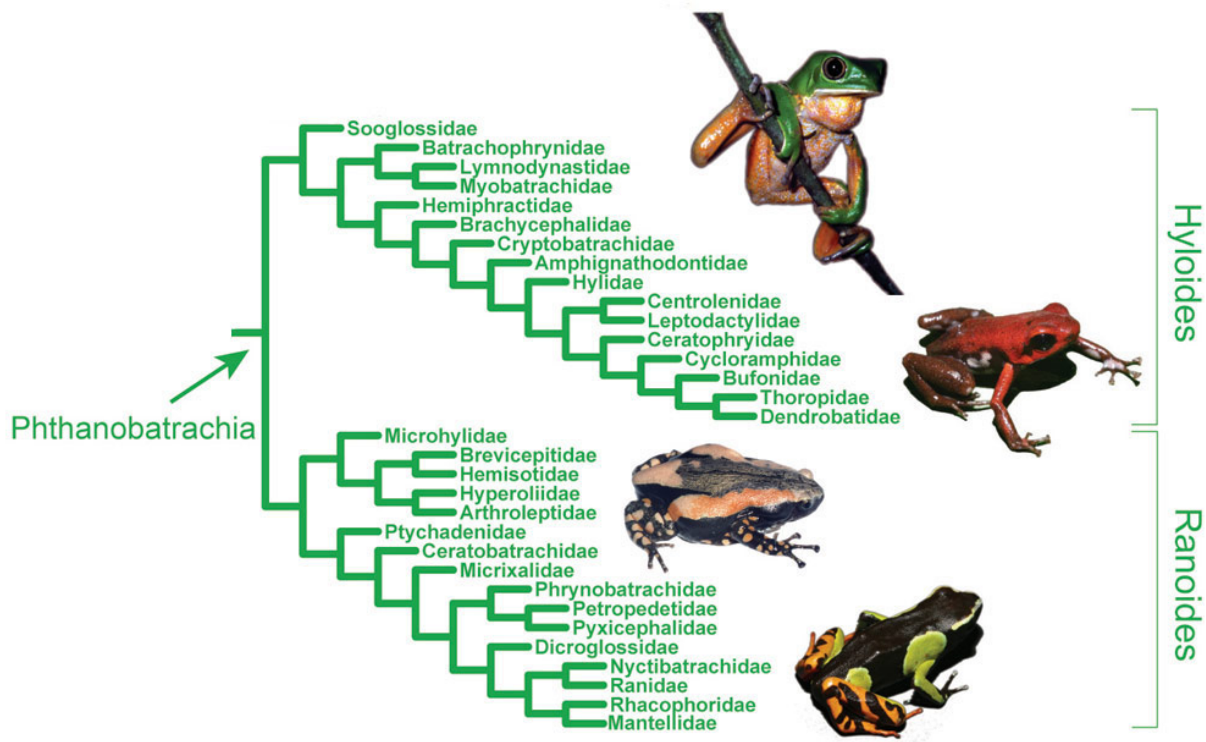
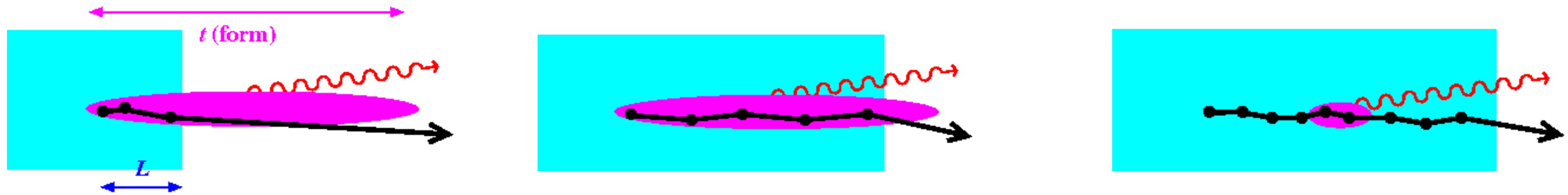


Taxonomy of Jet Quenching Formalisms

Does it handle thick or thin media, or both?

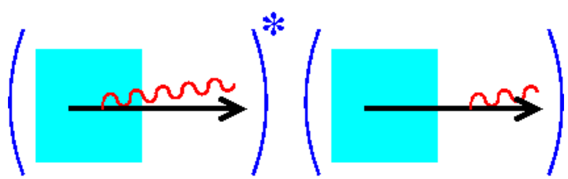


thin $L \ll t_{\rm form}$

thick $L \gtrsim t_{\rm form}$

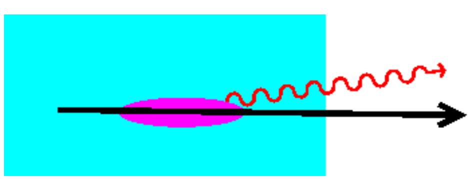
"infinite" $L \gg t_{\rm form}$

Must deal with medium-vacuum interference:

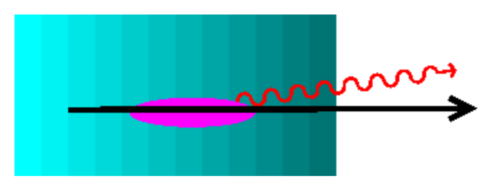


N/A

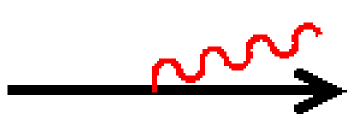
Does it handle non-uniform, time-dependent media?



versus

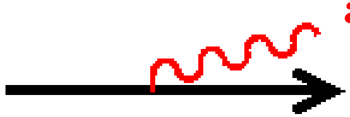


Does it only handle *soft* gluon bremsstrahlung?



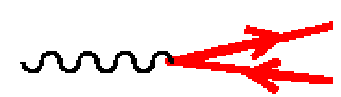
$x \ll 1$

versus

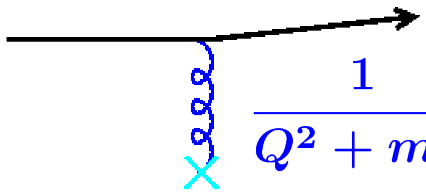


any x

+

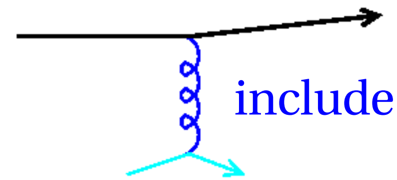


Does it assume "static" scatterers?



$$\frac{1}{Q^2 + m_{\mathrm{D}}^2}$$

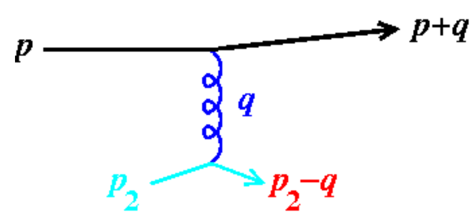
versus



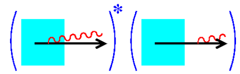
include magnetic forces

Does it include final-state Bose enhancement or Fermi blocking factors for plasma particles?

$$n\,\frac{d\sigma_{\mathrm{el}}}{d^2q_\perp} \qquad \text{versus} \qquad \frac{d\Gamma_{\mathrm{el}}}{d^2q_\perp} = \int dq_z \int d^3p_2\, \frac{d\sigma_{\mathrm{el}}}{d^3q}\, f(p_2)\, [1 \pm f(\vec{p}_2 - \vec{q})]$$

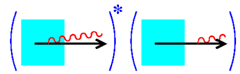


Issues on this page are relevant if you want to get exactly the correct answer in the weak coupling limit.

	thickness		non-uniform media?	x values	non-static scatterers and $1 \pm f$?	exact for small α?
Zakharov	any	yes	yes	any	no	no
BDMPS	any	yes	yes	any	no	no

BDMPS ('96)   Zakharov ('96)

BDMPS = Baier, Dokshitzer, Mueller, Peigne, Schiff

	thickness		non-uniform media?	x values	non-static scatterers and $1 \pm f$?	exact for small α?
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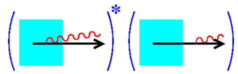
BDMPS ('96) Zakharov ('96)

equivalence ('98)

a problem in non-Hermitian 2-D quantum mech.

$$H(t) = \frac{p_\perp^2}{2\mathcal{M}} - i\,\Gamma(x_\perp, t)$$

	thickness		non-uniform media?	x values	non-static scatterers and $1\pm f$?	exact for small α?
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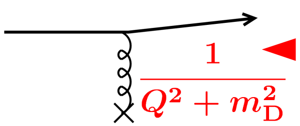
$\text{BDMPS}_{\text{model}}$	any	yes	yes	any	no	no



static

BDMPS ('96) Zakharov ('96)

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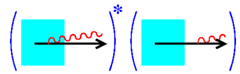
	thickness		non-uniform media?	x values	non-static scatterers and $1 \pm f$?	exact for small α?
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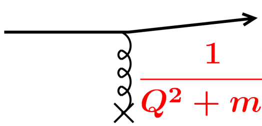
$\text{BDMPS}_{\text{model}}$	any	yes	yes	any	no	no
BDMPS_{HO}	thick	yes	yes	any	no	no



$\dfrac{1}{Q^2 + m_D^2}$

static

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$$H(t) = \frac{p_\perp^2}{2\mathcal{M}} - i\,\Gamma(x_\perp, t)$$

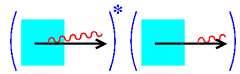
thick

$\ln(E/T) >> 1$

harmonic oscillator

($\hat{q}$ introduced)

$$H(t) = \frac{p_\perp^2}{2\mathcal{M}} - i\hat{q}x_\perp^2$$

	thickness		non-uniform media?	x values	non-static scatterers and $1 \pm f$?	exact for small α?
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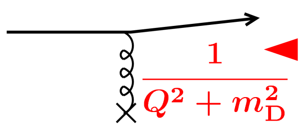
BDMPS$_{\text{HO}}$	thick	yes	yes	any	no	no
ASW	any/thin	yes	yes	$x << 1$	no	no



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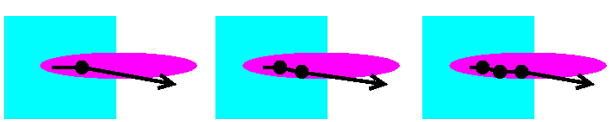
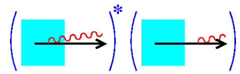
opacity expansion

GLV, ASW ('99-'03)



$x << 1$, static

GLV = Gyulassy, Levai, Vitev; ASW = Armesto, Salgado, Wiedemann

	thickness		non-uniform media?	x values	non-static scatterers and $1 \pm f$?	exact for small α?
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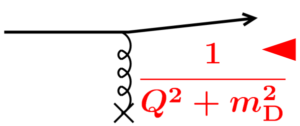
ASW	any/thin	yes	yes	$x << 1$	no	no
AMY	"infinite"	N/A	uniform	any	yes	yes



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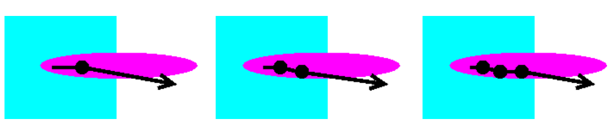


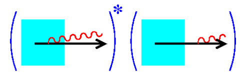
$x << 1$, static

AMY ('02-'03)

infinite media

exact for small α

AMY = Arnold, Moore, Yaffe

	thickness		non-uniform media?	x values	non-static scatterers and $1 \pm f$?	exact for small α?
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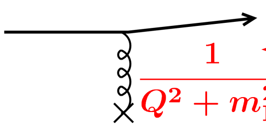
AMY	"infinite"	N/A	uniform	any	yes	yes
generalized BMDPS-Z	any	yes	yes	any	yes	yes



static

BDMPS ('96) Zakharov ('96)

equivalence ('98)

a problem in non-Hermitian 2-D quantum mech.

thick

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harmonic oscillator

($\hat{q}$ introduced)

thin

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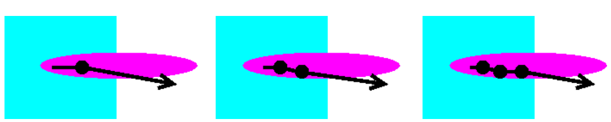


$x << 1$, static

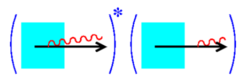
AMY ('02-'03)

infinite media

exact for small α

generalized BMPDS-Z $\rightarrow$ AMY ('09)

$$n\, d\sigma_{\text{el}} \rightarrow d\Gamma_{\text{el}}$$

	thickness		non-uniform media?	x values	non-static scatterers and $1\pm f$?	exact for small α?
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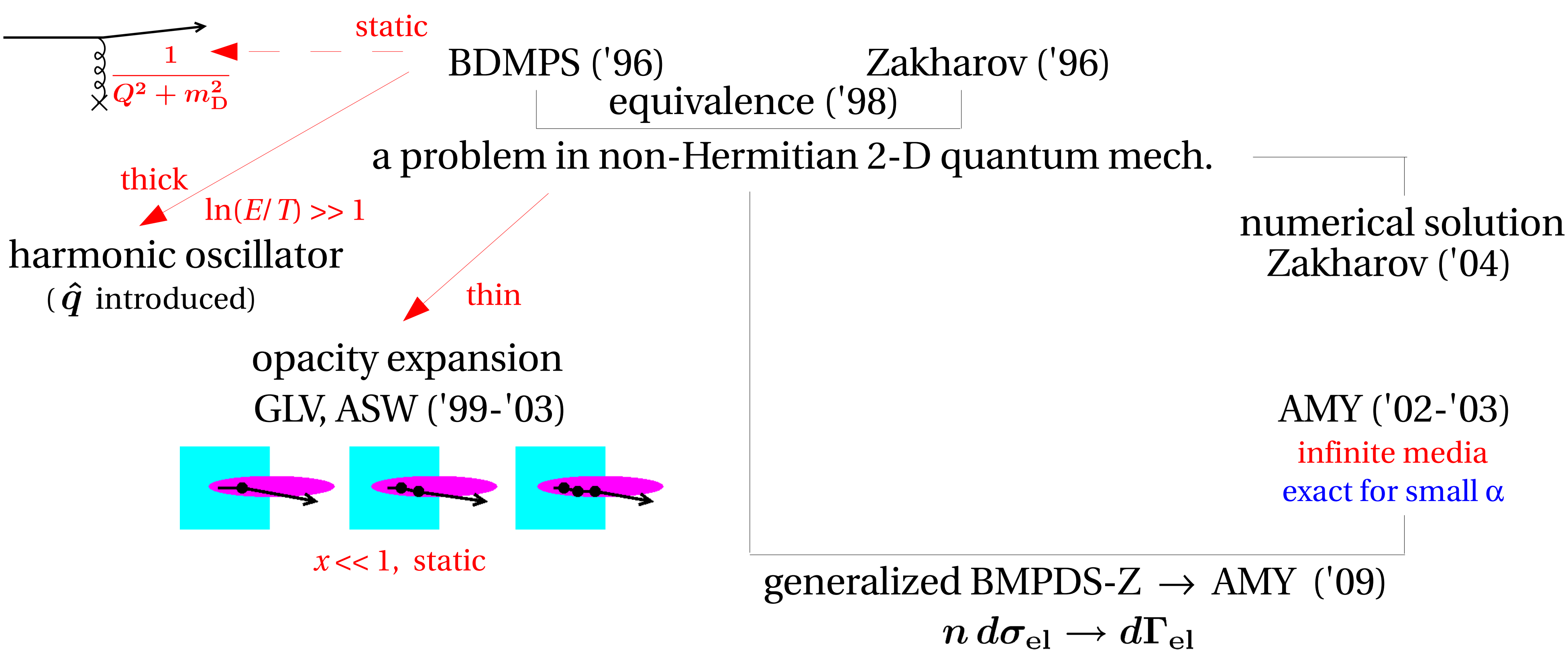
generalized BDMPS-Z	any	yes	yes	any	yes	yes



Summary

• Weak coupling ain't simple at high temperature – lots of rich, complicated physics.

• The LPM effect is easy to understand qualitatively!

• There's a simple generalization of earlier formalisms for calculating the LPM effect in QCD that will yield exact results in the weak coupling limit if one simply uses weak-coupling results for the elastic scattering rate $d\Gamma_{el}$.

Issues with weak coupling analysis

Practical issue: How big is the next-order correction in α_s?

result = (leading order) [1 + O(g)]

How big can α_s be before correction is 100% effect?

Example: $d^2\Gamma_{\rm el}/dq_\perp^2$ [Caron-Huot '09]

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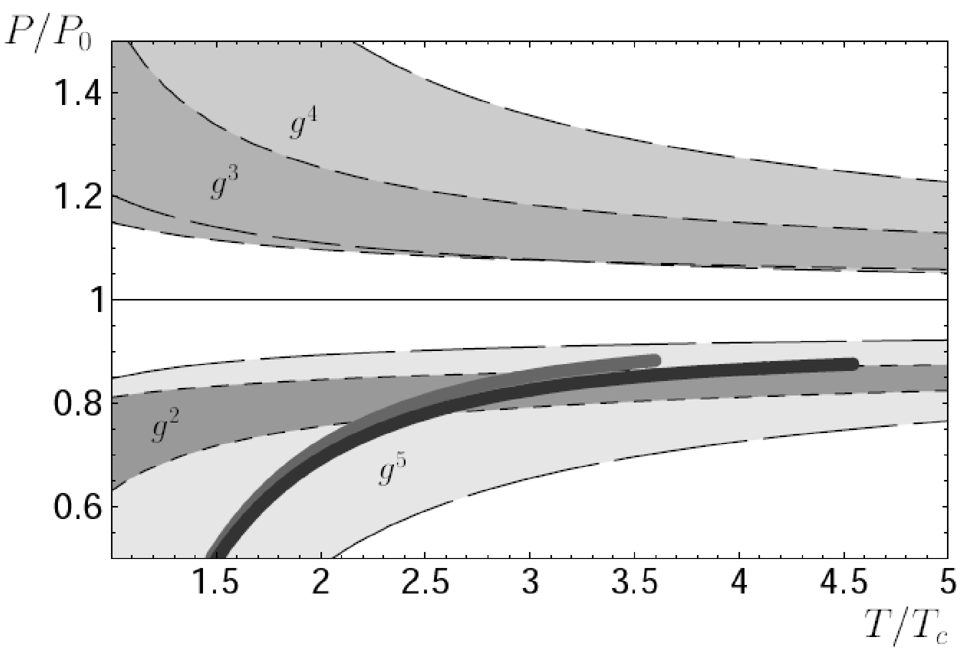
Similar to long-standing problem with the QCD equation of state:



g^2 and g^3 corrections the same size when

$\alpha_s \sim 0.1 \sim \alpha_s(100\ \mathrm{GeV})$

Folks have tried various resummations of perturbation theory...

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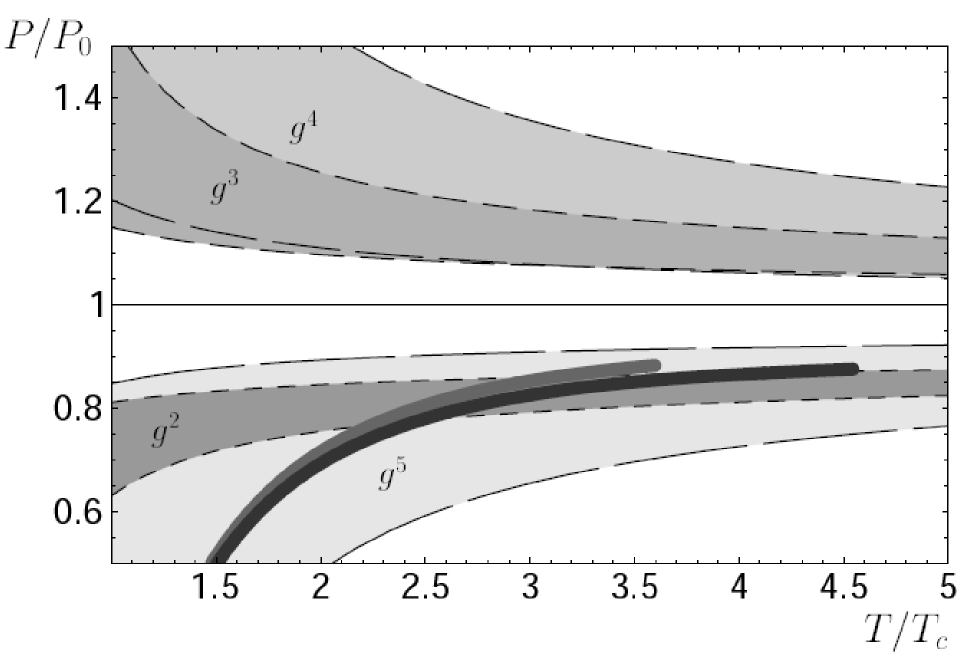

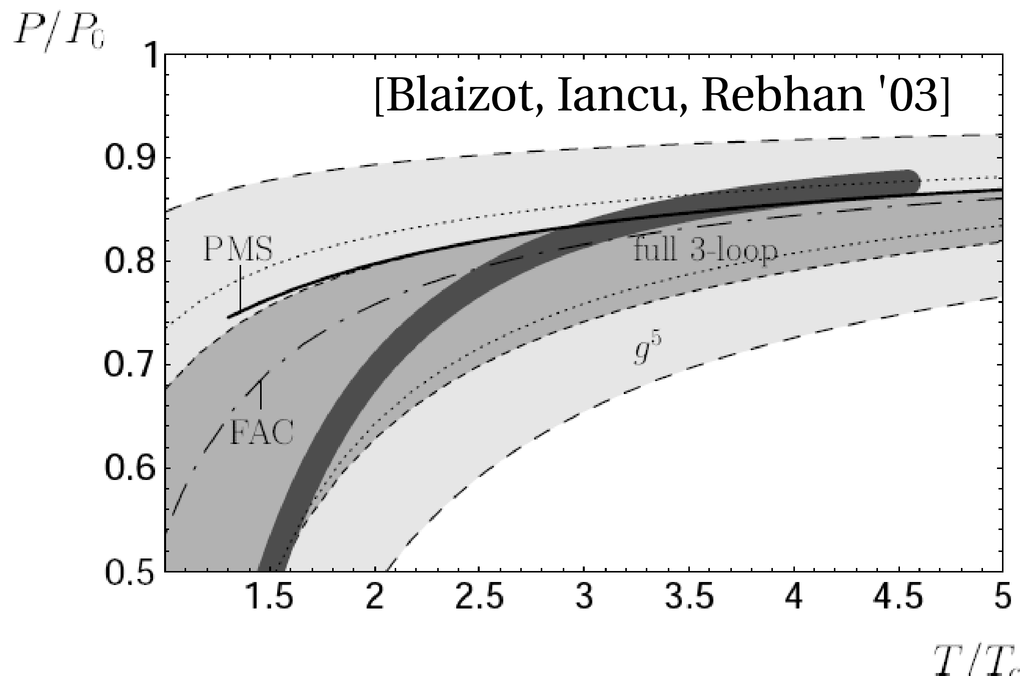

Not clear how to generalize to dynamics...

Issues with weak coupling analysis

Theoretical issue:

Weak coupling $\alpha_s(T) << 1$

If LPM effect ignored: stopping distance $\propto$ $\ln E$

Actual result: stopping distance $\propto$ $\left(\frac{E}{\ln E}\right)^{1/2}$

Strong coupling $\alpha_s \to \infty$ in large-N_c $\mathcal{N}$=4 SUSY QCD

stopping distance $\propto$ $E^{1/3}$

What's the first correction to the exponent for small α ?